



# Modelling of U-tube Tanks for ShipMo3D Ship Motion Predictions

*Kevin McTaggart*

*Prepared for:*

*Canadian Coast Guard Major Crown Projects Directorate  
Ottawa*

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## Defence R&D Canada – Atlantic

External Client Report

DRDC Atlantic ECR 2011-300

January 2012

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# Abstract

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Ship roll motions in waves can be significant due to small roll damping and the proximity of ship natural roll frequency to encountered wave frequencies. Roll motions can be reduced by various methods, including bilge keels and active roll stabilization using rudders or dedicated stabilizer fins. U-tube tanks and flume tanks can also be used to reduce roll motions, and have the advantages of being effective at zero ship speed and being protected from environmental hazards such as ice. This report describes the implementation of a U-tube tank model for the ShipMo3D ship motion library. The U-tube tank model has been implemented for computations in both the frequency and time domains. Example computations for a generic frigate demonstrate the reduction of roll motions using a U-tube tank. The relative effectiveness of a U-tube tank will be greatest when a ship has small roll damping, such as a vessel with small or no bilge keels.

# Résumé

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Le mouvement de roulis des navires dans les vagues peut être important à cause du faible amortissement du roulis et de la proximité de la fréquence naturelle de roulis du navire par rapport à la fréquence de roulis des vagues rencontrées. Le mouvement de roulis peut être réduit à l'aide de divers moyens, y compris l'installation de quilles de roulis et de dispositifs de stabilisation actifs, comme des gouvernails ou des dérives de stabilisation. Les citernes à tube en U et les citernes antiroulis peuvent également être utilisées pour réduire le mouvement de roulis ; elles ont l'avantage d'être efficaces même lorsque le navire est à l'arrêt et sont protégées contre les dangers associés à certaines conditions environnementales, comme la glace. Le présent rapport décrit la mise en œuvre d'un modèle de citerne à tube en U destiné à enrichir la bibliothèque de données sur le mouvement des navires du logiciel ShipMo3D. Le modèle de citerne à tube en U a été mis en œuvre pour effectuer des calculs dans le domaine temporel et le domaine fréquentiel. Des exemples de calculs pour une frégate générique démontrent la réduction du roulis lorsque l'on utilise une citerne à tube en U. L'efficacité relative d'une citerne à tube en U sera maximale lorsque le navire amortit faiblement le roulis, comme dans le cas des navires possédant une quille de roulis de petite taille ou n'en possédant aucune.

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# Executive summary

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## Modelling of U-tube Tanks for ShipMo3D Ship Motion Predictions

Kevin McTaggart; DRDC Atlantic ECR 2011-300; Defence Research and Development Canada – Atlantic; January 2012.

**Introduction:** Ship roll motions in waves can be significant due to small roll damping and the proximity of ship natural roll frequency to encountered wave frequencies. Roll motions can be reduced by various methods, including bilge keels and active roll stabilization using rudders or dedicated stabilizer fins. U-tube tanks and flume tanks can also be used to reduce roll motions, and have the advantages of being effective at zero ship speed and being protected from environmental hazards such as ice.

**Principal Results:** This report describes the implementation of a U-tube tank model for the ShipMo3D ship motion library. The U-tube tank model has been implemented for computations in both the frequency and time domains. Example computations for a generic frigate demonstrate the reduction of roll motions using a U-tube tank. The U-tube tank capability has been integrated into ShipMo3D Version 3.0.

**Significance of Results:** The ShipMo3D ship motion library can now be used to assess the influence of U-tube tanks on ship motions. Parametric studies can be performed, including investigation of the influence of tank dimensions and associated natural frequency on ship roll motions. Frequency domain predictions are computationally efficient and provide data that is readily usable. Time domain predictions are useful for examining nonlinear phenomena, such as when U-tube tank fluid motions and effectiveness are limited by the size of the tank.

**Future Plans:** The ShipMo3D ship motion library will be actively maintained and supported.

# Sommaire

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## Modelling of U-tube Tanks for ShipMo3D Ship Motion Predictions

Kevin McTaggart ; DRDC Atlantic ECR 2011-300 ; Recherche et développement pour la défense Canada – Atlantique ; janvier 2012.

**Introduction :** Le mouvement de roulis d'un navire dans les vagues peut être important en raison du faible amortissement et de la proximité de la fréquence naturelle de roulis du navire par rapport à la fréquence de roulis des vagues rencontrées. Le mouvement de roulis peut être réduit à l'aide de divers moyens, y compris l'installation de quilles de roulis et de dispositifs de stabilisation actifs, comme des gouvernails ou des dérives de stabilisation. Les citernes à tube en U et les citernes antiroulis peuvent également être utilisées pour réduire le mouvement de roulis ; elles ont l'avantage d'être efficaces même lorsque le navire est à l'arrêt et sont protégées contre les dangers associés à certaines conditions environnementales, comme la glace.

**Résultats principaux :** Le présent rapport décrit la mise en œuvre d'un modèle de citerne à tube en U servant à enrichir la bibliothèque de données sur les mouvements de navires du logiciel ShipMo3D. Le modèle de citerne à tube en U a été mis en œuvre pour effectuer des calculs dans le domaine temporel et le domaine fréquentiel. Des exemples de calculs pour une frégate générique démontrent la réduction du roulis lorsque l'on utilise une citerne à tube en U. La capacité des citernes à tube en U a été intégrée à la version 3.0 du logiciel ShipMo3D.

**Importance des résultats :** La bibliothèque de données sur les mouvements de navires du logiciel ShipMo3D peut maintenant être utilisée pour évaluer l'incidence des citernes à tube en U sur les mouvements de navires. Des études paramétriques, y compris des études sur l'incidence des dimensions de la citerne et de la fréquence naturelle connexe sur le mouvement de roulis des navires, peuvent être réalisées. Les prévisions dans le domaine fréquentiel sont efficaces sur le plan des calculs et fournissent des données qui sont directement utilisables. Les prévisions dans le domaine temporel sont, quant à elles, utiles pour examiner certains phénomènes non linéaires, comme c'est le cas lorsque le mouvement des fluides dans les citernes à tube en U et l'efficacité sont limités par les dimensions de la citerne.

**Travaux ultérieurs prévus :** La bibliothèque de données sur les mouvements de navires du logiciel ShipMo3D sera activement mise à jour et appuyée.



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# 1 Introduction

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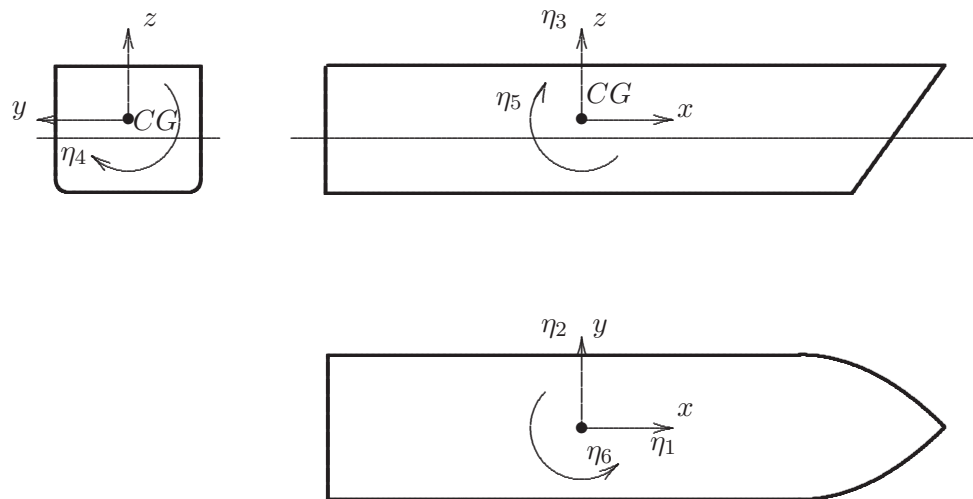
Ship motions in waves affect the performance of both humans and ship systems. Ship roll motions can be especially large because the natural roll frequency of a ship can be similar to encountered wave frequencies and ship roll damping can be quite small. When designing a ship, several steps can be taken to reduce roll motions in waves. Bilge keels are very effective and are widely used. Active roll stabilization using rudders or dedicated roll stabilizer fins can be effective if a ship is travelling with sufficient forward speed to generate required lift forces on rudders or fins. U-tube tanks and flume tanks can be used to reduce roll motions, and are effective at all ship speeds. Furthermore, U-tube tanks and flume tanks are rugged and can be contained within the hull, thus making them protected from ice and other environmental hazards that a ship might encounter.

In anticipation of ongoing design work, the Canadian Coast Guard tasked DRDC Atlantic with adding a capability for modelling U-tube tanks in the ShipMo3D ship motion library. U-tube tank modelling has been implemented in both the frequency and time domains in Version 3 of ShipMo3D [1, 2]. ShipMo3D Version 3 also includes modelling of sloshing in tanks with free surfaces [3]. The U-tube tank implementation is based on the work of Lloyd [4]. Section 2 gives coordinate systems used for evaluating ship motions in waves. The equations of motions for the ship and U-tube tank fluid are given in Section 3, with the numerical implementation described in Section 4. Section 5 gives an example design of a U-tube tank for a generic frigate. Final conclusions are given in Section 6.

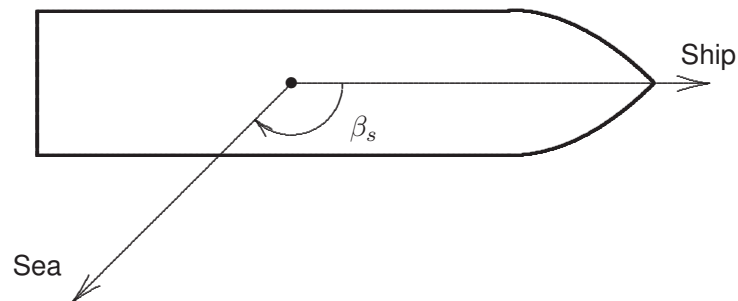
## 2 Coordinate Systems for Ship Motions in Waves

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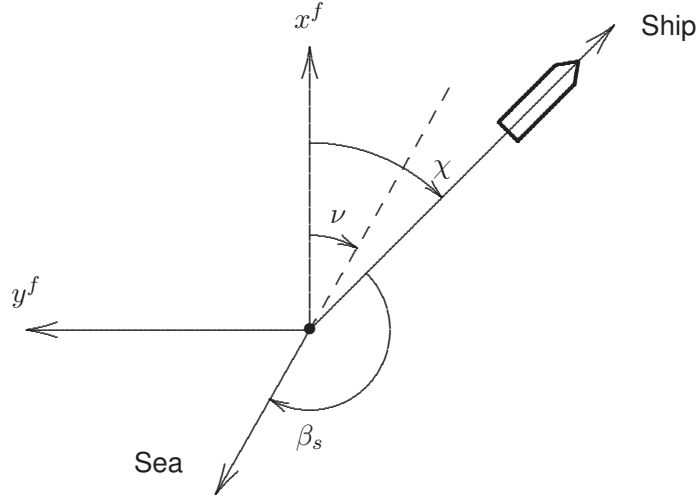
ShipMo3D uses 2 main coordinate systems for evaluating ship motions in waves. Figure 1 shows a translating earth coordinate system, which is used for frequency domain motions and also for evaluating hydrodynamic forces. The translating earth coordinate system moves with the mean forward speed of the ship. Figure 2 shows how relative sea direction is defined in the translating earth coordinate system. For time domain computations, which can involve a freely maneuvering ship, motions are given in the earth-fixed coordinate system of Figure 3. Within ShipMo3D time domain computations, forces and accelerations at each time step are evaluated using a translating earth coordinate system and then converted to the earth-fixed coordinate system.



**Figure 1:** Translating Earth Coordinate System for Frequency Domain Motions and Solution of Hydrodynamic Forces



**Figure 2:** Relative Sea Direction



**Figure 3:** Earth-Fixed Coordinate System for Time Domain Motions

### 3 Equations of Motion Including U-tube Tank

The equations of ship motion including a U-tube tank can be evaluated based on the work of Lloyd [4]. The equations presented here are based on the coordinate systems presented in Figures 1 and 4; thus, some terms differ from those in Reference 4 due to differences in coordinate sign conventions.

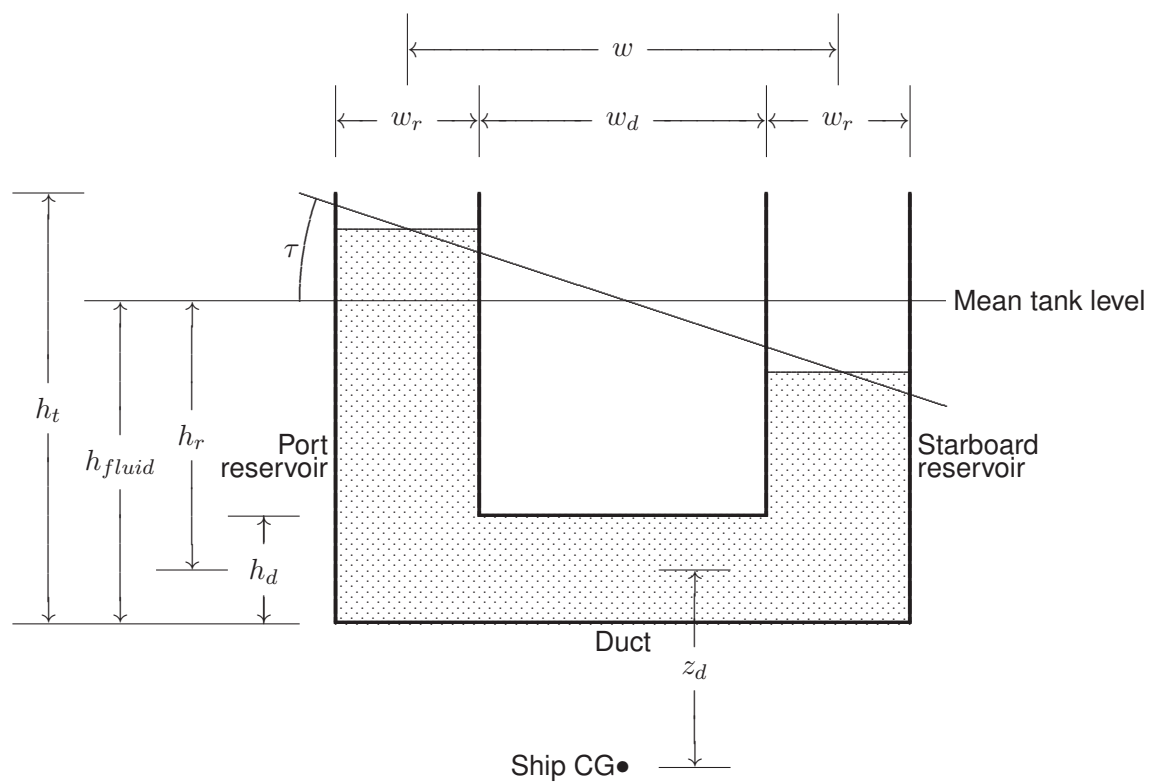
For a ship with 6 rigid-body degrees of freedom plus a degree of freedom  $\tau$  for the U-tube tank (see Figure 4), the equations of motion in the frequency domain can be written as follows:

$$\begin{aligned} \left[ \begin{array}{c|c} M_{\eta\eta}^e & M_{\eta\tau}^e \\ \hline M_{\tau\eta}^e & M_{\tau\tau}^e \end{array} \right] \left\{ \begin{array}{c} \ddot{\eta} \\ \ddot{\tau} \end{array} \right\} + \left[ \begin{array}{c|c} B_{\eta\eta} & 0 \\ \hline 0 & B_{\tau\tau} \end{array} \right] \left\{ \begin{array}{c} \dot{\eta} \\ \dot{\tau} \end{array} \right\} \\ + \left[ \begin{array}{c|c} C_{\eta\eta} & C_{\eta\tau} \\ \hline C_{\tau\eta} & C_{\tau\tau} \end{array} \right] \left\{ \begin{array}{c} \eta \\ \tau \end{array} \right\} = \left\{ \begin{array}{c} F_{\eta} \\ 0 \end{array} \right\} \end{aligned} \quad (1)$$

The matrices and vectors in the above equation are partitioned into ship motion and U-tube tank fluid angle portions, and the effective mass matrix  $[M^e]$  includes contributions from both ship mass and added mass. The following term is introduced:

$$Q_{tank} = \frac{1}{2} \rho_{tank} w_r w^2 L_{tank} \quad (2)$$

where  $\rho_{tank}$  is the density of fluid in the tank,  $w_r$  is the reservoir width,  $w$  is the nominal width between reservoirs, and  $L_{tank}$  is the longitudinal length of the tank.



**Figure 4:** Schematic of U-tube Tank



Considering first only the displacement of the tank fluid, the effective mass and stiffness terms are:

$$M_{\tau\tau}^e = Q_{tank} w_r \left( \frac{w}{2 h_d} + \frac{h_r}{w_r} \right) \quad (3)$$

$$C_{\tau\tau} = Q_{tank} g \quad (4)$$

where  $h_d$  is the height of the duct and  $h_r$  is the nominal height of the fluid in each reservoir relative to mid-height of the duct. The undamped natural frequency of the tank fluid displacement  $\tau$  is given by:

$$\omega_\tau = \sqrt{\frac{C_{\tau\tau}}{M_{\tau\tau}^e}} \quad (5)$$

The above equation can be re-expressed using the tank dimensions:

$$\omega_\tau = \sqrt{\frac{2 g h_d}{w_r w + 2 h_r h_d}} \quad (6)$$

The damping  $B_{\tau\tau}$  associated with the tank fluid displacement  $\tau$  is highly dependent on the local fluid flow within the tank, including viscous effects. In the present formulation, the damping  $B_{\tau\tau}$  is expressed in terms of a non-dimensional damping coefficient  $b_{\tau\tau}$ , which is the fraction of critical damping:

$$B_{\tau\tau} = \frac{2 b_{\tau\tau} Q_{tank} g}{\omega_\tau} \quad (7)$$

The damping coefficient  $b_{\tau\tau}$  can be estimated using physical model tests or computational fluid dynamics, with values typically being of the order of 0.10.

The ship motions influence the motion of the tank fluid, giving rise to the following terms:

$$M_{\tau 2}^e = Q_{tank} \quad (8)$$

$$M_{\tau 4}^e = Q_{tank} (-z_d + h_r) \quad (9)$$

$$M_{\tau 6}^e = Q_{tank} x_{tank} \quad (10)$$

$$C_{\tau 4} = Q_{tank} g \quad (11)$$

where  $z_d$  is the nominal elevation of the duct relative to the ship centre of gravity and  $x_{tank}$  is the longitudinal position of the tank relative to the ship centre of gravity.

The motion of the tank fluid influences the ship motions, as given by the following:

$$M_{2\tau}^e = -Q_{tank} \quad (12)$$

$$M_{4\tau}^e = Q_{tank} (-z_d + h_r) \quad (13)$$

$$M_{6\tau}^e = -Q_{tank} x_{tank} \quad (14)$$

$$C_{4\tau} = Q_{tank} g \quad (15)$$

It should be noted that the ship roll stiffness  $C_{44}$  does not include the effect of the U-tube tank; however, the effective reduction in ship metacentric height can be evaluated given the following limit for low frequency motion:

$$\tau = -\eta_4 \text{ for } \omega_e \rightarrow 0 \quad (16)$$

The resulting effective change in static metacentric height is

$$\Delta \overline{GM}^{tank} = -\frac{Q_{tank}}{\Delta} \quad (17)$$

where  $\Delta$  is the ship mass displacement. As discussed later, the above effective change in metacentric height is included in ShipMo3D computations with a U-tube tank.

The fluid motion in the tank will be limited by the vertical extent of the tank, which should be considered during both design and time domain analysis. The maximum fluid displacement is:

$$\tau_{max} = \arctan\left(\frac{h_t - h_{fluid}}{w/2}\right) \quad (18)$$

## 4 Implementation within ShipMo3D

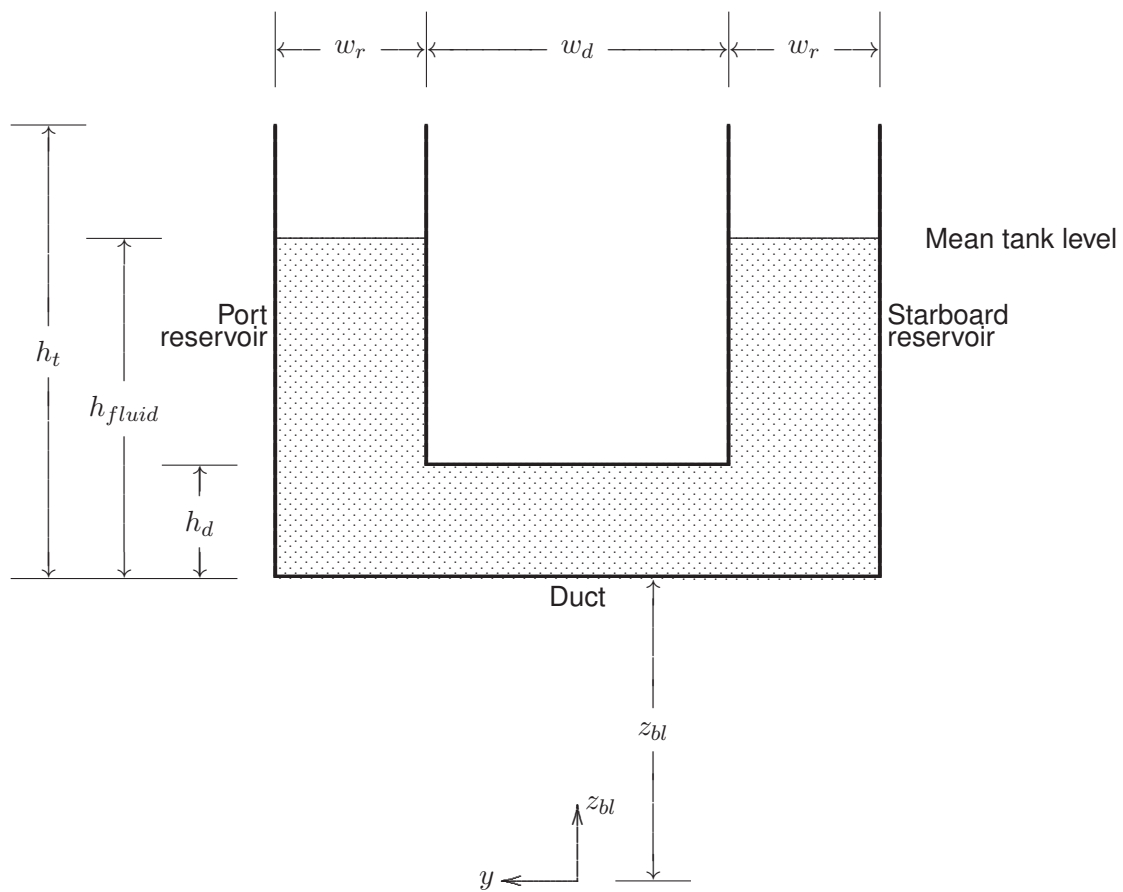
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ShipMo3D Version 3 has been completed, including user manuals [1, 2] and verification and validation [5]. ShipMo3D Version 3 introduces the following capabilities:

- modelling of U-tube tanks as described in this report,
- modelling of sloshing in tanks with free surfaces [3],
- the application SM3DSeakeepSeawayFromRaos for fast computation of ship motions in waves using previously computed response amplitude operators [2].

Figure 5 shows some of the U-tube tank input dimensions for the application SM3DBuildShip of ShipMo3D Version 3. In addition to modifications to SM3DBuildShip, the ShipMo3D applications SM3DFreeMo, SM3DSeakeepRegular, SM3DSeakeepRandom, and SM3DSeakeepSeaway have been modified to include modelling of U-tube tanks.

SM3DBuildShip includes an input record for `correctionGM`, the correction to metacentric height  $\overline{GM}$ . This term can be used to account for effects such as internal tanks that contain liquid. Note that the input for a U-tube tank in Version 3 of SM3DBuildShip accounts for the dynamic effects of the U-tube tank; thus, `correctionGM` should not include effects of U-tube tanks for which full input has been entered.



**Figure 5:** ShipMo3D Input for U-tube Tank

Within ShipMo3D frequency domain computations, the solution of ship motions including a U-tube tank is done according to Equation (1), in which ship and U-tube tank motions are considered to be fully coupled and are solved simultaneously. When solving for ship and U-tube tank motions in the time domain, minor simplifications are used such that the ship and U-tube tank motions are solved separately. The following is assumed when solving for ship motions in the time domain:

$$\ddot{\tau}(t + \Delta t) \approx \ddot{\tau}(t) \quad (19)$$

where  $\Delta t$  is the time step increment for solving ship motions. The above assumption is likely acceptable provided that  $\Delta t$  is small or the inertia forces acting on the ship due to the U-tube tank are small.

The motions of the U-tube tank fluid are solved in the time domain based on Equation (1):

$$\ddot{\tau} = \frac{1}{M_{\tau\tau}} (-[M_{\tau\eta}^e] \{\ddot{\eta}\} - [B_{\tau\eta}] \{\dot{\eta}\} - B_{\tau\tau} \dot{\tau} - [C_{\tau\eta}] \{\eta\} - C_{\tau\tau} \tau) \quad (20)$$

When performing time domain computations of the fluid displacement angle within a U-tube tank, consideration must be given to the limiting maximum displacement angle  $\tau_{max}$  imposed by tank geometry. If the limiting maximum displacement angle will be exceeded in a given time step due to the acceleration given by Equation (20), then the acceleration is modified such that the limit displacement angle will not be noticeably exceeded:

$$\ddot{\tau}(t) = \epsilon_{tau} \frac{\tau_{max} - \tau(t) - \dot{\tau}(t)\Delta t}{1/2 (\Delta t)^2} \text{ for } \tau(t) > 0 \quad (21)$$

$$\ddot{\tau}(t) = \epsilon_{tau} \frac{-\tau_{max} - \tau(t) - \dot{\tau}(t)\Delta t}{1/2 (\Delta t)^2} \text{ for } \tau(t) < 0 \quad (22)$$

where  $\epsilon_{tau}$  is an acceleration attenuation factor for maintaining stability during time domain simulations, with 0.2 being a recommended value.

## 5 Computations for Generic Frigate with U-Tube Tank

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Computations have been performed based on the generic frigate of the ShipMo3D user manuals [1, 2]. These sample computations demonstrate the design of a U-tube tank, and also provide code verification.

## 5.1 Design of U-tube Tank

Guidance for design of U-tube tanks is provided by Lloyd [4], Gawad et al. [6], and Moaleji and Greig [7]. For optimal dynamic performance, the natural frequency of the tank fluid (Equation (5)) should be approximately the same as the natural roll frequency of the ship. To limit problems with loss of static stability, the effective reduction in metacentric height (Equation (17)) should be less than 25 percent of the ship metacentric height. Past experience indicates that a U-tube tank will have a mass of 1-5 percent of the ship mass. Dynamic performance is generally best if the tank is at a similar or higher elevation than the ship centre of gravity.

Table 1 gives the properties of the generic frigate. Using these properties, the design of the U-tube tank can be done in the following steps:

1. The tank is located at midships. The bottom of the tank is located 3 m above the baseline (i.e.,  $z_{bl} = 3$  m), and the tank has a total height  $h_t$  of 6 m. The vertical dimensions of the tank enable large fluid motions and also place the tank in a favourable position relative to the centre of gravity. The total width of the tank  $w_d + 2w_r$  is 12 m, taking advantage of the width available to maximize effectiveness for available volume. The reservoir widths  $w_r$  are assigned values of 2 m, leading to a duct width  $w_d$  of 8 m and reservoir spacing  $w$  of 10 m. To maximize the allowable fluid displacement, the mean fluid level  $h_{fluid}$  is set to  $(h_t + h_d)/2$ , which can be shown mathematically to give  $h_r = 0.5h_t$  (i.e.,  $h_r = 3$  m).
2. The effective allowable change in metacentric height  $\Delta \overline{GM}^{tank}$  is specified to have a nominal value of  $-0.2$  m (20 percent of a projected metacentric height of 1.0 m with future ship growth). The parameter  $Q_{tank}$  has a nominal limit of 743000 kg m (Equation (2)), and the tank length  $L_{tank}$  can be set to 7 m (Equation (2)).
3. The tank natural roll period is specified to have a nominal value equal to the ship natural roll frequency of 0.704 rad/s. Based on Equation (6), a duct height  $h_d$  of 0.6 m can be used to give a tank natural frequency of 0.706 rad/s.
4. The mass of the tank fluid is evaluated to be 126 tonnes, 3 percent of the ship mass, which is considered to be acceptable.

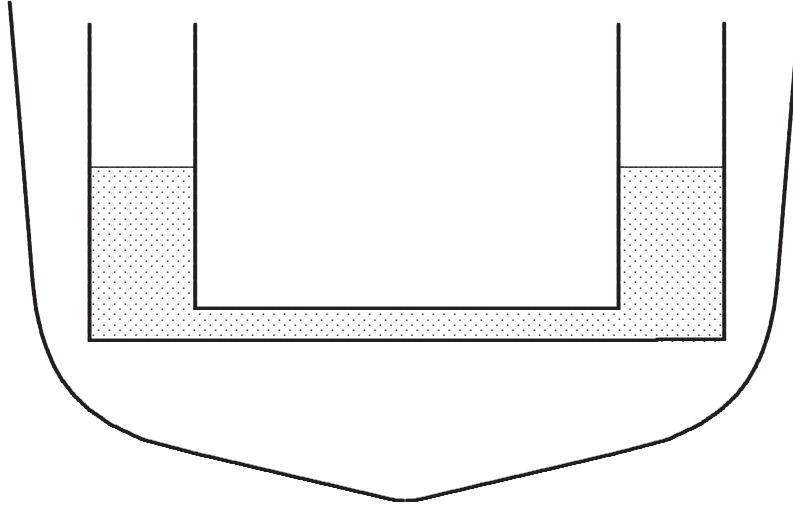
Table 2 gives the properties of the example tank, and Figure 6 shows the U-tube tank and the hull section at midships.

**Table 1:** *Generic Frigate Properties*

Length between perpendiculars, $L_{pp}$	120.0 m
Draft at midships, $T_{mid}$	4.2 m
Trim by stern, $t_{stern}$	0.0 m
Displacement, $\Delta$	3713 tonnes
Height of CG above baseline, $\overline{KG}$	6.0 m
Metacentric height, $\overline{GM}$	1.43 m
Natural roll frequency, $\omega_4$	0.704 rad/s
Breadth at midships	15.0 m
Hull depth at midships	9.4 m

**Table 2:** *Properties of Example U-tube Tank for Generic Frigate*

Fluid density (fresh water)	1000 kg/m <sup>3</sup>
Station (AP at station 20)	10
Length, $L_{tank}$	7.0 m
Duct width, $w_d$	8.0 m
Reservoir width, $w_r$	2.0 m
Height of bottom above baseline, $z_{bl}$	3.0 m
Total height, $h_t$	6.0 m
Duct height, $h_d$	0.6 m
Fluid height, $h_{fluid}$	3.3 m
Fluid mass, $m_{fluid}$	126 tonnes
Tank fluid frequency, $\omega_{tau}$	0.706 rad/s
Maximum fluid angle, $\tau_{max}$	28.4°
Effective change in metacentric height, $\Delta\overline{GM}^{tank}$	-0.189 m



**Figure 6:** Example U-tube Tank for Generic Frigate

## 5.2 Verification of U-tube Tank Time Domain Computations with Prescribed Motions

Numerical tests using prescribed motions were performed to verify that the time domain implementation of the U-tube tank is working correctly. All tests were based on the example U-tube tank with properties given in Table 2.

A decay test examined fluid displacement angle after an initial fluid displacement of 10 degrees. The U-tube tank itself was stationary (i.e., not subjected to ship motion). Figure 7 shows a time series from the decay test. Many standard references, including Reference 8, give equations for analyzing dynamic properties based on time series. The damping coefficient  $b_{\tau\tau}^t$  based on the time series was computed using:

$$b_{\tau\tau}^t = \frac{\log(\tau_i^{max}/\tau_{i+n}^{max}) \omega_{\tau}^{damped}}{2 \pi n \omega_{\tau}} \quad (23)$$

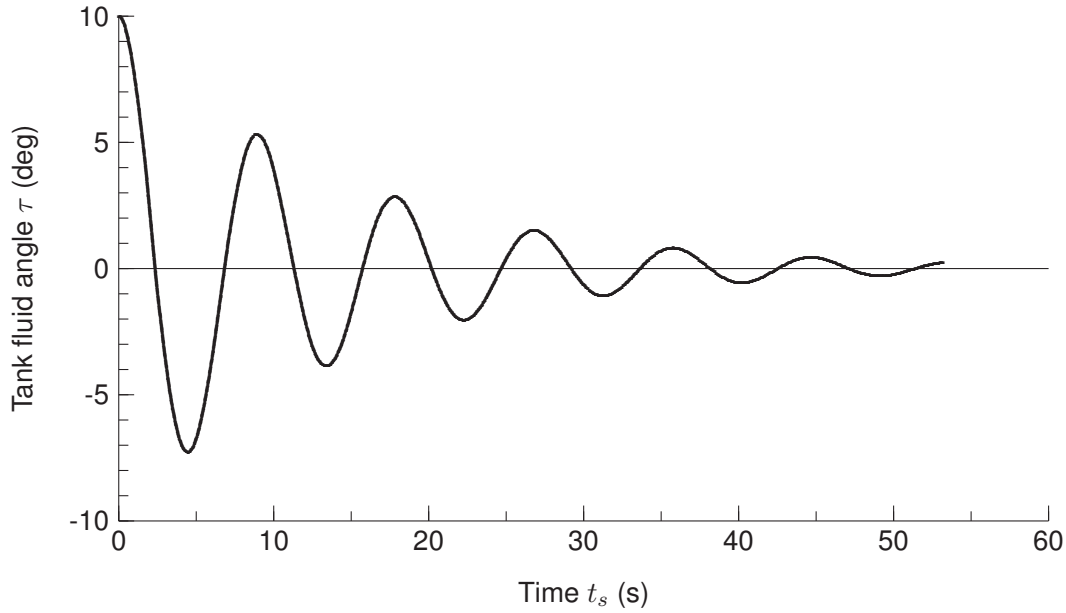
where  $\tau_i^{max}$  is the local maximum fluid angle after  $i$  cycles,  $n$  is the number of time series cycles for analysis, and  $\omega_{tau}^{damped}$  is the damped natural frequency given by:

$$\omega_{tau}^{damped} = \omega_{\tau} \sqrt{1.0 - b_{\tau\tau}^2} \quad (24)$$

The natural frequency based on the time series was computed using

$$\omega_{\tau}^t = \frac{2 \pi n}{(t_{i+n}^{max} - t_i^{max}) \sqrt{1.0 - b_{\tau\tau}^2}} \quad (25)$$

where  $t_i^{max}$  is the time for local maximum  $i$ .



**Figure 7:** Time Series of U-tube Tank Fluid Angle after Initial Displacement Angle of 10 degrees

Table 3 compares the actual tank natural frequency and damping coefficient with values obtained from analysis of the decay time series. The values obtained from the decay time series are in excellent agreement with the actual natural frequency and damping coefficient for the U-tube tank.

**Table 3:** Comparison of U-tube Tank Natural Frequency and Damping with Values Determined by Decay Time Series

	Natural frequency $\omega_\tau$	Damping coefficient $b_{\tau\tau}$
Actual	0.706 rad/s	0.100
Decay time series	0.707 rad/s	0.100

Two additional numerical tests considered the U-tube tank with an initial fluid displacement angle of zero and the tank subjected to the following ship roll motion:

$$\eta_4(t) = 0 \text{ for } t < 0 \quad (26)$$

$$\eta_4(t) = \hat{\eta}_4 [1.0 - \exp(t/t_c)] \text{ for } t \geq 0 \quad (27)$$

where  $\hat{\phi}_4$  is the roll amplitude and  $t_c$  is a time constant. Figures 8 and 9 show time series for ship roll and fluid displacement angle when the time constant  $t_c$  has a value



of 10 s and the roll amplitude has values of 15 degrees and 40 degrees respectively. Figure 8 shows the fluid displacement angle approaches a value of  $-15$  degrees ( $-\hat{\eta}_4$ ), as expected. In Figure 9, the ultimate fluid displacement angle is limited by the physical limits of the tank, which impose a limit of  $-28.4$  degrees. The time series for the large roll angle indicates that the time domain computations can manage fluid displacement angles at the limits of the tank.

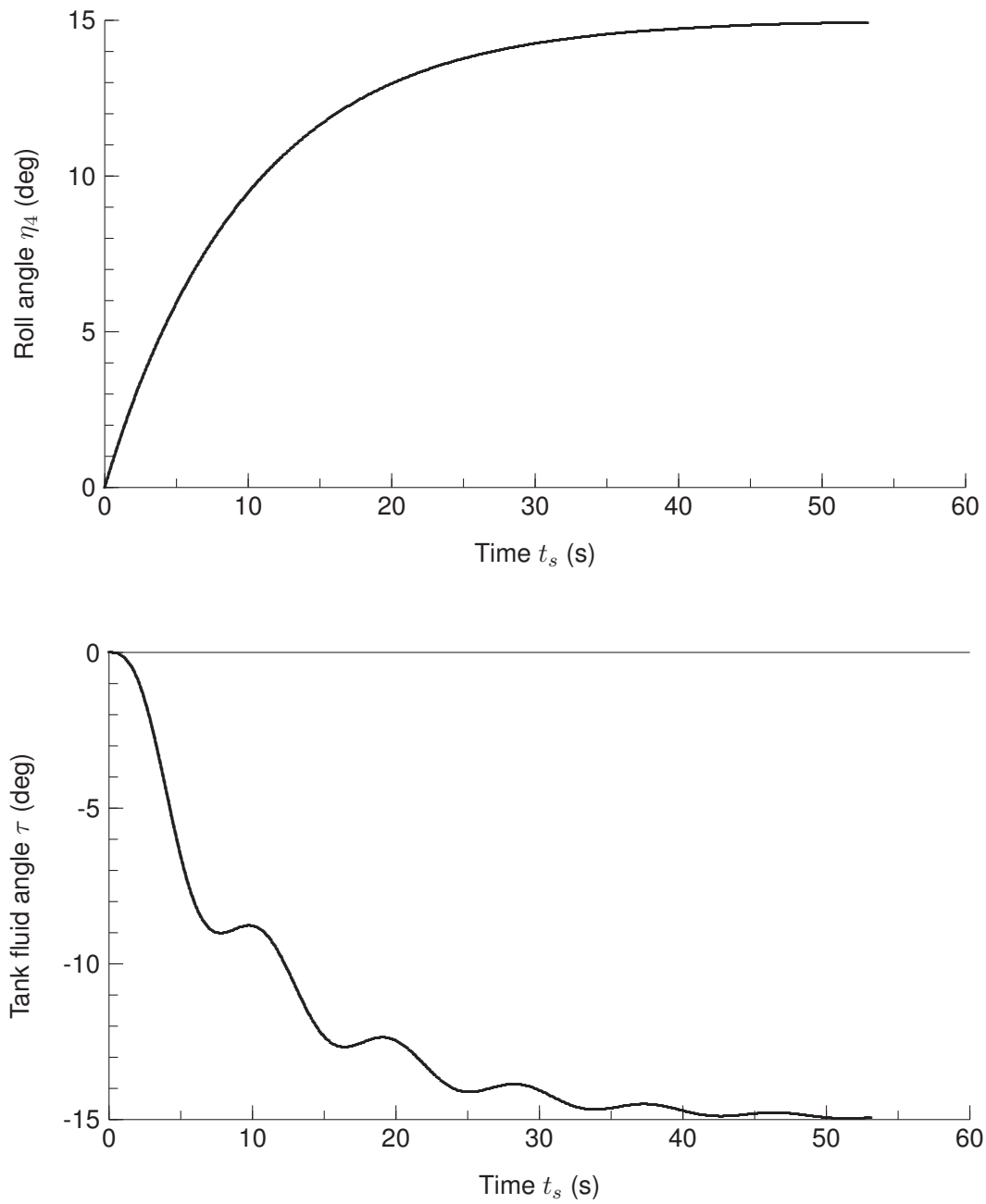
### 5.3 Verification of Ship Motion Computations

Motion computations in both the time and frequency domains were performed using the generic frigate fitted with and without a U-tube tank. Figures 10 and 11 show roll RAOs and U-tube tank fluid angle RAOs for the generic frigate travelling at 10 knots in beam seas with wave steepnesses of 0.01 (1/100). This low wave steepness was selected so that the U-tube tank fluid motion wouldn't be limited by the top of the tank, thus facilitating comparison between frequency and time domain results. The plotted roll motions show that the U-tube tank reduces roll amplitude when the wave encounter frequency is near the ship natural roll frequency. The trends in Figure 10 are very similar to those presented for an example by Lloyd [4]. The plotted results show excellent agreement between computations in the time and frequency domains.

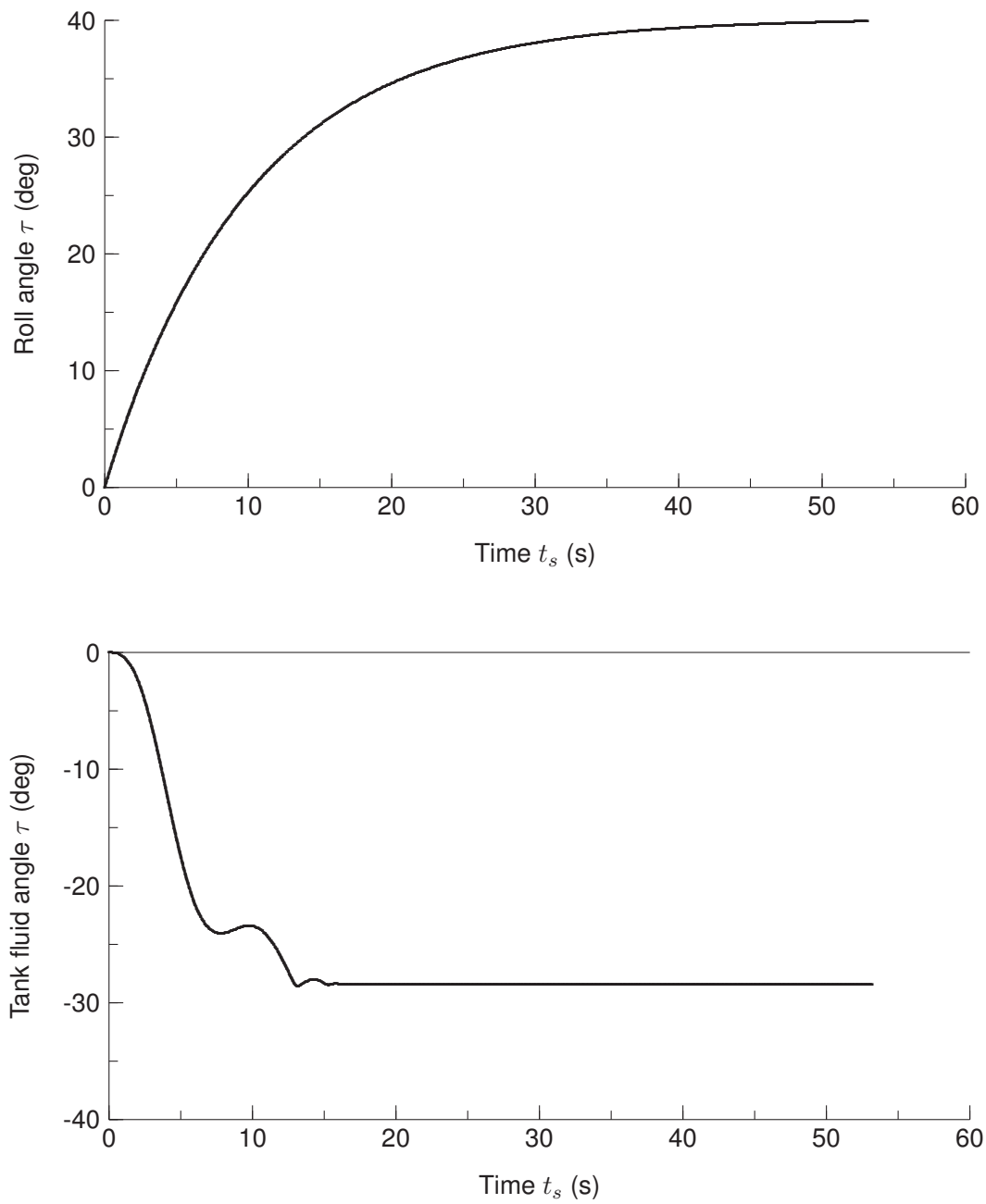
Figures 12 and 13 show RMS roll and fluid displacement for the generic frigate travelling at 10 knots in sea state 5, which is modelled by a long-crested seaway and a Bretschneider spectrum with significant wave height of 3.25 m and peak wave period of 9.7 s. The U-tube tank reduces roll motions at most headings, and provides the greatest benefit when the encountered waves have a period near the natural roll frequency of the ship. It should be noted that the generic frigate has substantial bilge keels to provide roll damping. For a ship fitted with smaller or no bilge keels, the relative reduction in roll motions provided by the U-tube tank would be significantly greater. The plotted results show excellent agreement between computations in the time and frequency domains for the ship in random seas.

### 5.4 Sensitivity of Motions to U-tube Tank Damping Coefficient

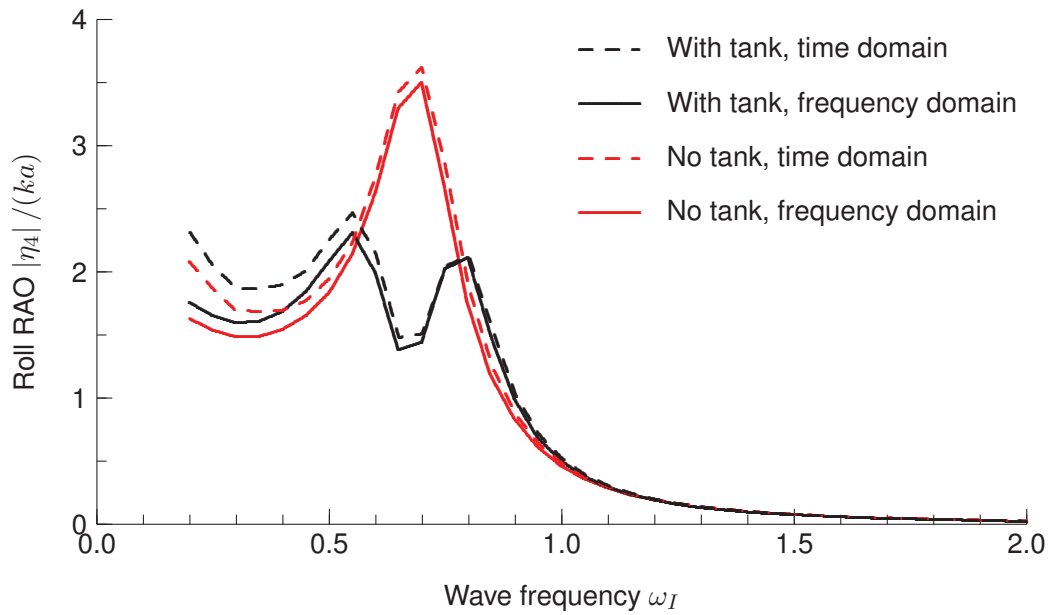
When modelling a U-tube tank, there is usually uncertainty when selecting a value for the input damping coefficient  $b_{\tau\tau}$ . Figure 14 shows frequency domain computations of ship roll motions using tank damping coefficients of 0.05, 0.10, and 0.20. The results indicate that ship roll motions are not very sensitive to input tank damping coefficients varying within a realistic range.



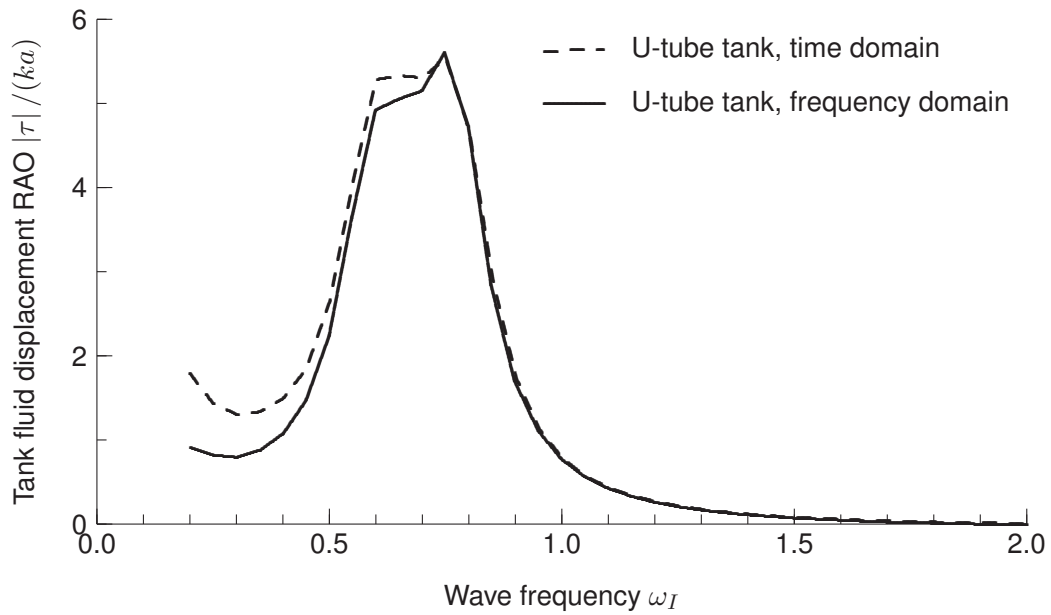
**Figure 8:** Time Series of Roll Motion and Fluid Tank Displacement for Ship Roll  $\eta_4 = 15^\circ [1.0 - \exp(-t/10 \text{ s})]$



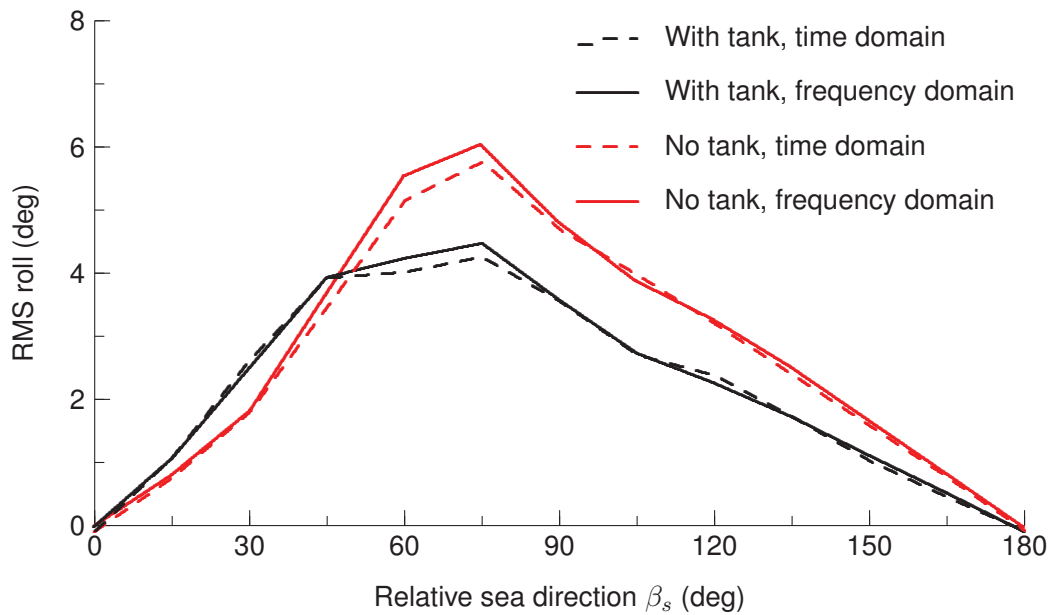
**Figure 9:** Time Series of Roll Motion and Fluid Tank Displacement for Ship Roll  $\eta_4 = 40^\circ [1.0 - \exp(-t/10 \text{ s})]$



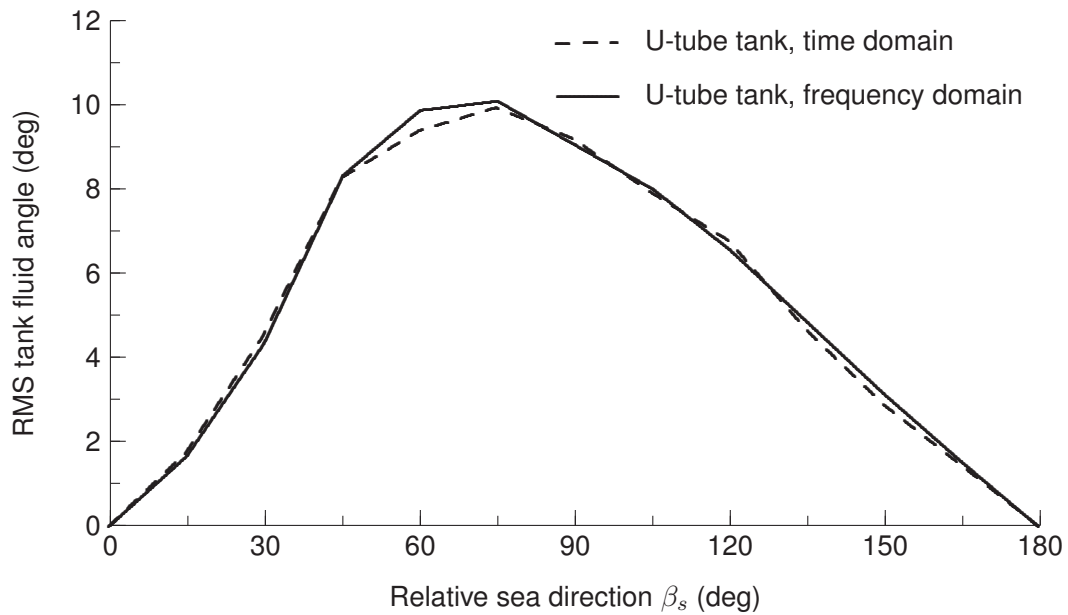
**Figure 10:** Roll Amplitude Versus Wave Frequency, Generic Frigate at 10 knots in Regular Waves, Beam Seas with Steepness of 0.01



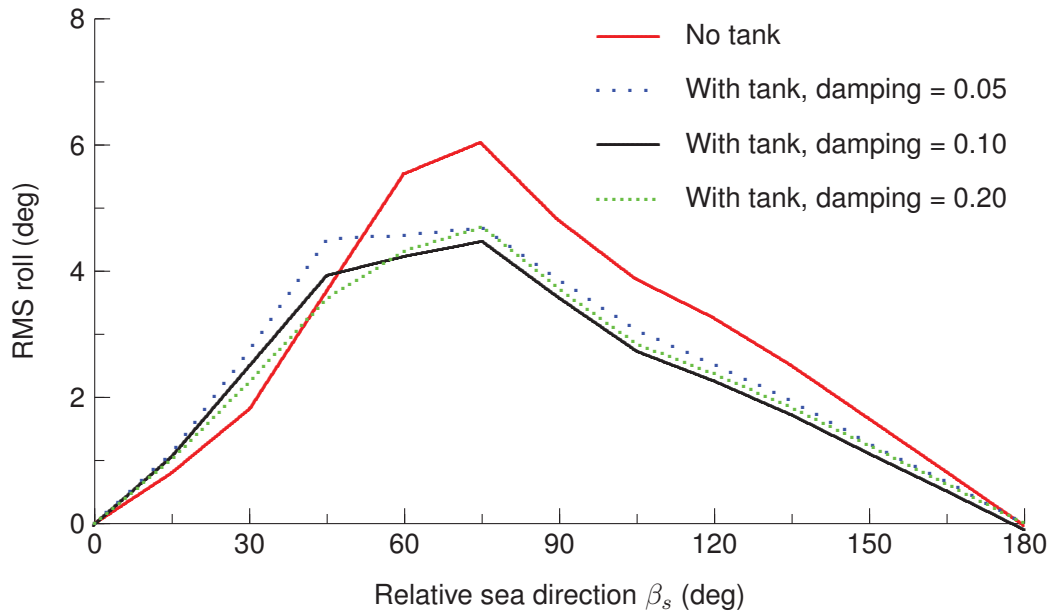
**Figure 11:** U-tube Tank Fluid Displacement Amplitude Versus Wave Frequency, Generic Frigate at 10 knots in Regular Waves, Beam Seas with Steepness of 0.01



**Figure 12:** RMS Roll Versus Relative Sea Direction, Generic Frigate at 10 knots in Sea State 5



**Figure 13:** RMS U-tube Tank Fluid Displacement Angle Versus Sea Direction, Generic Frigate at 10 knots in Sea State 5



**Figure 14:** RMS Roll Versus Relative Sea Direction, Generic Frigate at 10 knots in Sea State 5, Different U-tube Tank Damping Coefficients

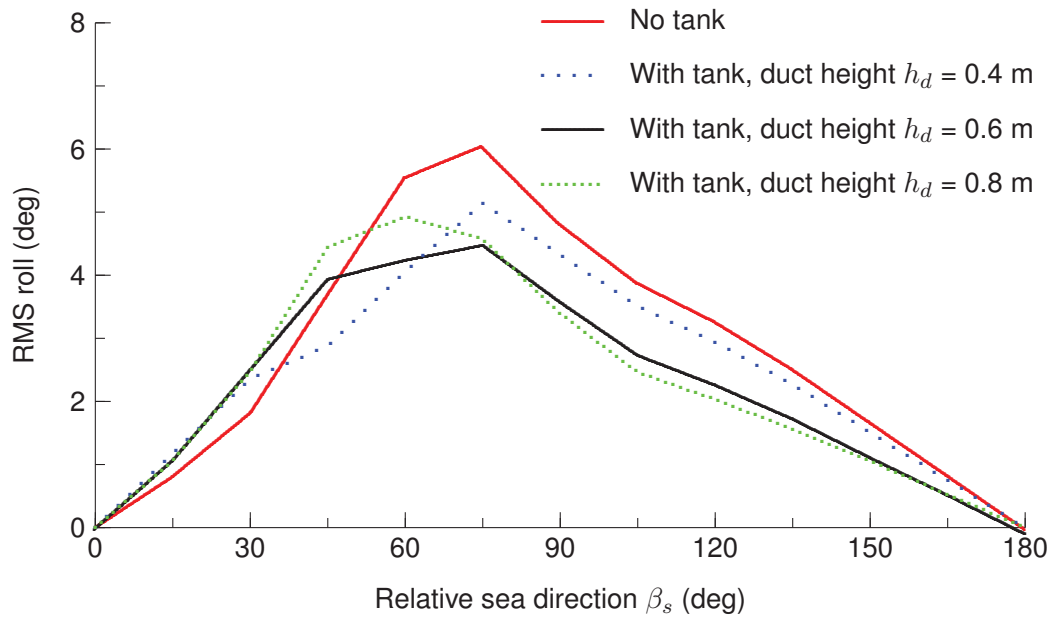
## 5.5 Sensitivity of Ship Motions to U-tube Tank Duct Height and Natural Frequency

The sensitivity of ship motions to U-tube tank natural frequency has been examined by varying the duct height  $h_d$  for the tank specified in Table 2. The height of fluid  $h_{fluid}$  is set to  $(h_d + h_t)$  for each given duct height. Table 4 gives the tank natural frequencies for each duct height, with the lowest and highest natural frequency being within 20 percent of the ship roll natural frequency.

Frequency domain computations given in Table 15 indicate that the roll reduction is generally greatest when the tank natural frequency is near the natural roll frequency of the ship.

**Table 4:** U-tube Tank Natural Frequencies for Varying Duct Heights

Duct height	Natural frequency
$h_d$	$\omega_{tau}$
0.4 m	0.592 rad/s
0.6 m	0.706 rad/s
0.8 m	0.795 rad/s



**Figure 15:** RMS Roll Versus Relative Sea Direction, Generic Frigate at 10 knots in Sea State 5, Different U-tube Tank Duct Heights

## 6 Conclusions

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Modelling of U-tube tanks has been implemented into the ShipMo3D ship motion library. Example computations for a generic frigate fitted with a U-tube tank exhibit expected motion behaviour and excellent agreement between computations in the time and frequency domains. Uncertainty typically exists regarding selection of an input damping coefficient for a U-tube tank; however, example computations indicate that ship motions aren't very sensitive to variation of the damping coefficient within a realistic range of values. Example computations show that a U-tube tank is most effective when its natural frequency is near the ship natural roll frequency.



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# Symbols and Abbreviations

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$a$	wave amplitude
$[B]$	damping matrix
$b_{\tau\tau}$	non-dimensional damping of U-tube tank
$b_{\tau\tau}^t$	non-dimensional damping of U-tube tank from time series analysis
$[C]$	stiffness matrix
CG	centre of gravity
$g$	gravitational acceleration
$H_s$	significant wave height
$h_d$	U-tube tank duct height
$h_{fluid}$	height of fluid in U-tube tank relative to tank bottom
$h_r$	nominal fluid height in U-tube tank reservoir
$h_t$	total height of U-tube tank
$k$	wave number
$L_{pp}$	ship length between perpendiculars
$L_{tank}$	longitudinal length of U-tube tank
$[M^e]$	effective mass matrix
$m_{fluid}$	mass of U-tube tank fluid
$n$	number of cycles for time series analysis
$Q_{tank}$	U-tube tank fluid inertia term
$RAO$	response amplitude operator
$T_{mid}$	ship draft at midships
$t_c$	roll motion time constant
$t_{stern}$	ship trim by stern
$t_i^{max}$	time for local maximum $i$
$w$	nominal distance between U-tube tank reservoirs
$w_r$	U-tube tank reservoir width
$x, y, z$	translating earth coordinates

$x_f, y_f$	earth-fixed horizontal plane coordinates
$x_{tank}$	longitudinal position U-tube tank relative ship CG
$z_{bl}$	height above baseline
$z_d$	nominal elevation of U-tube tank duct relative to ship CG
$\beta_s$	relative sea direction
$\Delta \overline{GM}^{tank}$	U-tube tank effective influence on ship metacentric height
$\Delta t$	time step for solving ship motions
$\epsilon_{\tau}$	U-tube tank acceleration attenuation factor
$\eta_j$	motion displacement for mode $j$ in translating-earth coordinates
$\hat{\eta}_4$	roll motion amplitude
$\nu$	wave heading (from)
$\rho_{tank}$	density of fluid in U-tube tank
$\tau$	U-tube tank fluid displacement angle
$\tau_{max}$	U-tube tank maximum possible fluid displacement angle
$\ddot{\tau}$	U-tube tank fluid acceleration
$\chi$	ship heading (to)
$\omega_4$	ship roll natural frequency
$\omega_\tau$	natural frequency of U-tube tank fluid displacement
$\omega_{\tau}^{damped}$	damped natural frequency of U-tube tank
$\omega_\tau^t$	natural frequency of U-tube tank from time series analysis
$\Delta$	ship mass displacement

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Ship roll motions in waves can be significant due to small roll damping and the proximity of ship natural roll frequency to encountered wave frequencies. Roll motions can be reduced by various methods, including bilge keels and active roll stabilization using rudders or dedicated stabilizer fins. U-tube tanks and flume tanks can also be used to reduce roll motions, and have the advantages of being effective at zero ship speed and being protected from environmental hazards such as ice. This report describes the implementation of a U-tube tank model for the ShipMo3D ship motion library. The U-tube tank model has been implemented for computations in both the frequency and time domains. Example computations for a generic frigate demonstrate the reduction of roll motions using a U-tube tank. The relative effectiveness of a U-tube tank will be greatest when a ship has small roll damping, such as a vessel with small or no bilge keels.

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